

PARAMETRIC EPICYCLIC RESONANCE IN BLACK HOLE DISKS: QPOs IN MICRO-QUASARS

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Abstract

Non-linear acoustic coupling of modes in accretion disks allows a parametric resonance between epicyclic motions. In black hole disks, such a resonance first occurs, and is strongest, when the radial and vertical epicyclic frequencies are approximately in a 2:3 ratio, in agreement with the 300 and 450 Hz frequencies reported in GRO J1655-40 and the 184 and 276 Hz frequencies reported in XTE J1550-564. The first overtone in the same resonance is in a 5:3 ratio with the fundamental, in agreement with the 69.2 and 41.5 Hz frequencies reported in GRS 1915+105. The narrow width and the frequency stability of the corresponding QPOs follow from general properties of parametric resonance.

Introduction

We give an explanation for the presence in black hole X-ray sources of the observed stable frequencies, manifest as narrow-width QPOs (quasi-periodic oscillations in the light curve). These oscillations arise as a result of parametric resonance in the accretion disk, and the actually observed rational ratios of frequencies correspond to that resonance which is strongest *a priori*.

Different modes in the disk are coupled through terms involving derivatives of enthalpy. Oscillations of pressure in one mode lead to harmonic variations of the eigenfrequency in another mode. In the limit of small pressure corrections, particular ratios of the epicyclic frequencies correspond to the condition for parametric resonance, leading to exponential growth of one of the modes. This is a specific mechanism through which the previously suggested (Kluźniak and Abramowicz 2001a,b; Abramowicz and Kluźniak 2001) resonant origin of high frequency QPOs in neutron stars and black holes can come about. For a discussion of the relation of this phenomenon to “quantization of orbits” see Abramowicz et al 2002. Further, we point out that coherent localized structures may form in the accretion disk.

Strohmayer (2001a) reports 300 and 450 Hz QPOs in GRO J1655-40, and we have already pointed out that these are in a 2:3 ratio (Abramowicz and Kluźniak 2001, Kluźniak and Abramowicz 2001a,b). Strohmayer 2001b reports 41.5 ± 0.4 Hz and 69.2 ± 0.15 Hz in GRS 1915+105, giving 0.599 as the frequency ratio, rather close to 3:5=0.6 (if taken at face value). These ratios seemed to us to be indicative of a resonant phenomenon, and the recent discovery of another pair of commensurate frequencies (in a 2:3 ratio), 184 and 276 Hz in XTE J1550-564 (Remillard et al. 2002; see also Miller et al. 2001), greatly strengthens the case for resonance.

Localized structures in the disk

We find, that “geostrophic” flow is a solution of the equations of disk hydrodynamics and this would correspond to a localized elliptic flow pattern, counter-rotating with the radial epicyclic frequency in the co-moving fluid frame. We point out that close to the inner edge of a thin accretion disk around black holes (and neutron stars), the vorticity becomes very small, and goes to zero where the radial epicyclic frequency vanishes, at the marginally stable orbit. It is, therefore, not difficult to excite a vorticity perturbation of large *relative* amplitude in that region. This is in contrast with the Newtonian case of a $1/r$ potential, for which Spiegel and coworkers find that an initial amplitude above a finite threshold is necessary for a vorticity perturbation to persist in the disk (Bracco et al., 1998).

Such localized structures may execute “up and down” motions away from the disk plane, and it follows from the equations of hydrodynamics (acceleration is equal to the gradient of the gravitational potential plus the gradient of enthalpy), that such motion is harmonic with frequency

$$\omega_2^2 = \Omega_z^2 + \epsilon,$$

while the corresponding harmonic motion in the radial direction occurs at frequency

$$\omega_1^2 = \Omega_r^2 + \gamma^2 \epsilon,$$

where Ω_r is the radial epicyclic frequency, Ω_z the vertical epicyclic frequency, ϵ , assumed to be $\ll 1$, is the pressure correction and γ^2 is the ratio of vertical to radial extent of the disturbance. The equation for ω_1 can be thought of as the high, or low, frequency limit of the general dispersion relation of diskoseismology (Wagoner 1999, Kato 2000), $\epsilon = c_s^2 k^2$ corresponds to the (“acoustic”) p -mode, while a negative value of ϵ corresponds to the (“internal gravity”) g -mode. We share the view stressed by diskoseismologists (e.g., Wagoner, Silbergleit, and Ortega 2001) that the very stability of the observed frequencies, while the system changes its luminosity (and presumably its accretion rate), implies that pressure forces are a minor perturbation.

Parametric resonance

We are trying to explain how QPOs may arise from minor fluctuations in radial motion. There is a mechanism which leads to large amplitudes of oscillatory motion, given small perturbations. It is parametric resonance. In the case under discussion, radial motions at frequency ω_1 will disturb the pressure at the same frequency, i.e., will induce harmonic variations in the value of ϵ . Then, the eigenfrequency of vertical oscillations, ω_2 , will itself undergo small harmonic variations in its value, occurring at frequency $\omega_1 \approx \omega_r$. This is precisely a description of parametric resonance in an oscillator with the equation of motion $d^2z/dt^2 + \omega_2^2[1 + h \cos(\omega_1 t)]z = 0$. Resonance occurs when $\omega_1 = 2\omega_2/n$, with n integer (Landau and Lifschitz 1973). For ω_1 in a small, and decreasing with n , range about $2\omega_2/n$, exponential growth of the amplitude of oscillations occurs until non-linear saturation. If a dissipation term is present, growth occurs only if a certain threshold value is exceeded by the amplitude, h , of variations in the value of the oscillator parameter.

The relatively large amplitude vertical oscillation can then in turn amplify radial oscillations through the usual resonance mechanism in driven oscillators.

Frequency ratios

Under the assumption (justified above) that $\omega_1 \approx \Omega_r$, and $\omega_2 \approx \Omega_z$, we note that parametric resonance in black hole accretion disks occurs only for $n \geq 3$. This is simply because in the Kerr metric, $\Omega_r < \Omega_z$, always, and the condition for parametric resonance is $\omega_1 = 2\omega_2/n$. The fastest growing resonant mode in black hole disks occurs when $\Omega_r = 2\Omega_z/3$, i.e., for the lowest possible value of n , when the two epicyclic frequencies are in a 2:3 ratio, because in general the lower the value of n , the faster the growth of amplitude. It is remarkable that in at least two black hole sources this is the ratio of the observed frequencies. We interpret these observed frequencies as the fundamental frequency of vertical oscillations excited through parametric resonance by oscillations at the radial epicyclic frequency, at 2/3 the value of the fundamental.

In non-linear resonances, combination frequencies appear in addition to the fundamental. In the case of a parametric 2:3 resonance, the first “combination” overtone appears at 5/3 the value of the fundamental. The 69.2 Hz frequency reported in GRS 1915+105 is indeed 5/3 the value of the also reported 41.5 Hz. This is highly suggestive. We note that under this interpretation, if the mass of the source is indeed as given by Greiner et al. (2001), i.e., it is no more than 18 or $20M_\odot$, this source would have to be a Schwarzschild black hole, or the spin parameter would have to have small negative values, i.e., the disk would be counter-rotating. In any case the magnitude of j would be much less than for the other two sources, GRO J1655-40 and XTE J1550-564.

A subharmonic at the frequency 1/3 of the fundamental would also be expected, and this has also been reported, for XTE J1550-564 (Remillard 2002).

Conclusions

Parametric resonance between epicyclic frequencies in a thin-disk accretion flow in the Kerr metric can lead to the excitation of finite amplitude oscillations and this can account for the appearance of commensurate “stable” frequencies in the observed sources. In addition to the strongest oscillations in the 2:3 ratio,

subharmonics and overtones are predicted, yielding a sequence of primes 1:2:3:5, all of which have been observed (albeit in various sources).

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